Creep Rupture Criteria for Salt

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ABSTRACT

The necessity for considering the possibility of failure as a result of time-dependent deformation is pointed out. It is shown that creep rupture is a mode of failure that is likely to occur in rocks which undergo substantial time-dependent deformation. Theoretical concepts with regard to creep rupture are discussed. Formulas for creep rupture under uniaxial conditions are developed; these are generalized for multiaxial stress conditions. Experimental data obtained from creep tests on rock salt are presented. On the basis of the theoretical review, which is supported by the limited experimental data available, it may be concluded that the controlling stress-time to rupture relationship can be represented as a straight line on a logarithmic plot. For multiaxial states of stress, there is some ambiguity about selecting the controlling stress. It has been suggested that for the conditions adjacent to the face of an excavation as represented by the triaxial extension test, the controlling stress is the maximum principal stress difference. It has been the practice to look for tensile stresses in rock surrounding an underground cavity as a potential source of trouble. It appears that the occurrence of tensile strain is also a critical factor. In a triaxial extension test, the applied stresses are all compressive; however, the resulting axial strain, which is tensile, causes the specimen to fail. Similar conditions are likely to occur at the face of underground excavations and could cause surface spalling. The methodology for combining creep rupture criteria with methods of stress analysis to evaluate the stability of an excavation is presented. A linear cumulative damage theory is utilized as it offers the only practical means of evaluating the potential of creep rupture under varying stress.

INTRODUCTION

In evaluating the possibility of failure in the design of openings in rock masses the governing criteria have, in general, been exclusively based on the assumption of the time-independent behavior of rock masses. However, certain rock types (e.g., rock salt) undergo substantial creep (time-dependent) deformation. A possible consequence of creep deformation in materials subjected to stresses, strains, or temperature is failure through creep rupture. This is a time-dependent phenomena. It has been hypothesized, Orowen (1949) that rupture occurs as a consequence of the sliding of grain boundaries. It has also been suggested that transcrystalline slip is the cause of rupture at high stresses. This type of phenomenon may occur in an experimental specimen or in any structure of service. Under conditions of creep, the magnitude of the failure stress depends essentially on the duration of the applied stress or strain. For high stresses, rupture occurs after a short time, and for low stresses the time before failure can be quite substantial. The estimate of the time to failure (creep rupture) under various stress conditions is of great importance in the practical design of structures undergoing creep. After having determined the appropriate creep rupture criteria for a particular material, it is necessary to combine these with methods of stress analysis to evaluate the stability of underground excavations. The methodology for doing this is outlined in this paper.

THEORY

In practical rock mechanics problems, materials which are, in general, not homogeneous and isotropic are subject to multiaxial, time-dependent, and nonhomogeneous stress states. Creep rupture theories which satisfactorily account for all these factors have not yet been developed. However, the available theories in conjunction with appropriate approximations can be used in practice. Theories for creep rupture were first developed for uniaxial, homogeneous and constant stress conditions. From this development, generalizations to include the more complicated stress states have been made. The discussion to follow will first briefly consider creep rupture for uniaxial stress states and then discuss their application for more general conditions.

Uniaxial state of stress

Hoff's theory of ductile creep rupture. One of the first attempts toward a phenomenological theory of ductile creep rupture was presented by Hoff (1953). Hoff considered the stretching of a cylindrical rod subject to a constant load, P. The length of the rod is denoted by L, and A is the area of cross-section (current values). For the initial conditions (i.e., at $t = 0$), we have $L = L_o$ and $A = A_o$. From the conditions of incompressibility, $LA = L_oA_o$. Hoff assumed that the strain rate* $\dot{\epsilon}$,

defined as $\frac{1}{A}\frac{dA}{dT}$

is related to the stress σ by a power law:

$$\dot{\epsilon} = B\sigma^n = B(P/A)^n \tag{1}$$

where B and n are constants. A solution of the above linear differential equation results in the following expression:

$$A^n - A_o^n = nP^nBt = nBA_o^n(\sigma_o)^nt \tag{2}$$

The time to failure **, t_H is determined from the assumption that the area will decrease to zero at failure. It is obvious that the area A becomes zero when:

$$t_H = \frac{1}{nB\sigma_o^n} \tag{3}$$

where σ_o is the initial stress; i.e., $\sigma_o = P/A_o$. In logarithmic coordinates:

$$\log t_H = -(\log n + \log B) - n \log \sigma_o = C - n \log \sigma_o \tag{4}$$

This is a straight line which relates the time to rupture t_H and the initial stress σ_o. The stress at any time t is related to the initial stress by the following expression:

$$\sigma = \sigma_o(1 - t/t_H)^{-1/n} \tag{5}$$

Brittle creep rupture (Kachanov's theory). Recognizing that brittleness in material will cause them to rupture before the cross-section shrinks to zero, both Hoff (1959) and Kachanov (1958) proposed methods to account for the deterioration of the material which is the cause of brittleness and results ultimately in rupture. Hoff's work was primarily empirical. Kachanov (1958, 1961) however, proposed a theoretical method for determining the time to brittle creep rupture. Kachanov attributed creep failure to the process of crack formation spreading against a background of growing creep deformations.

The deterioration with time is expressed in the assumption that the area of the sample supporting the load is less than the actual area of the sample and decreases with time. This decrease in area not only includes the effect of stretching of the sample, but also the effects of crack formation and local failures. To express this deterioration, Kachanov introduced a factor ψ, which is called the continuity of the material and is defined as:

$$0 \leq \psi = \frac{A_r}{A} \leq 1 \tag{6}$$

Where A is the actual current area of the sample and A_r is that area which supports the load. An alternative measure called the damage factor D defined as

$$\frac{A - A_r}{A}$$

has been used by Odqvist (1966). Equation 6 indicates that the stress σ_r on the actual supporting area A_r is larger than the current value of the mean stress σ which is equal to P/A.

It is assumed that the decrease in continuity with time can be expressed by equation:

$$\frac{d\psi}{dt} = f(\sigma_r) = f(\sigma/\psi) \tag{7}$$

$\psi = 1$ for time $t = o$. When the material ruptures at time*** t_{kr}, $\psi = o$. It is further assumed that the above equation is separable, therefore,

$$f\left(\frac{\sigma}{\psi}\right)$$

must be of the form $g(\sigma)\,h(\psi)$.
Hence,

$f\left(\frac{\sigma}{\psi}\right)$ can be expressed as:

$$f(\sigma/\psi) = -C(\sigma/\psi)^v \tag{8}$$

The minus sign in the above equation indicates the decrease of ψ with time. The value $\frac{\sigma}{\psi}$ may be interpreted as the "effective" stress acting on the specimen. Using the initial condition that at time $t = o$, $\psi = 1$, and substituting the expression for $f\left(\frac{\sigma}{\psi}\right)$ from Equation 8, in Equation 7, the following result is obtained on integrating Equation 7:

$$1 - \psi^{v+1} = C(1 + v)\int_o^t \sigma^v dt \tag{9}$$

Assume that **** t_{kc} represents the time to rupture under a constant stress σ_k. Utilizing Equation 9 and recognizing that $\psi = 0$ at rupture, the time to rupture (t_{kc}) for a constant stress σ_k is given by the following expression:

*This is based on the "logarithmic" strain
**The subscript H is used to identify Hoff's theory.
***The subscript k is used to identify Kachanov's theory.
****The subscript c is used to identify constant stress conditions.

$$t_{kc} = \frac{1}{C(1 + \upsilon)\,\sigma_k^{\upsilon}} \tag{10}$$

Equation 10 recast into logarithmic form is as follows:

$$\log t_{kc} = \log \frac{1}{C(1 + \upsilon)} - \upsilon \log \sigma_k \tag{11}$$

This indicates that on a logarithmic plot, the relationship between σ_k and t_{kc} can be represented by a straight line.

It is observed from Equations 4 and 11, that the stress-time to rupture relation for both the brittle and ductile rupture theories plot as straight lines on a logarithmic plot. Their relative positions for practical values of the parameters n, B, C and υ are shown in Figure 1.

Utilizing Equation 10, Equation 9 may be rewritten as:

$$1 - \psi^{\upsilon + 1} = \int_0^t \left(\frac{\sigma}{\sigma_k}\right)^{\upsilon} \frac{dt}{t_{kc}} \tag{12}$$

At rupture $t = t_{kr}$ $\psi = 0$; therefore, to determine the time to rupture of a material for a variable stress, the following equation must be solved for t_{kr}:

$$1 = \int_0^{t_{kr}} \left(\frac{\sigma}{\sigma_k}\right)^{\upsilon} \frac{dt}{t_{kc}} \tag{13}$$

Combined ductile and brittle creep rupture. Kachanov, in an attempt to describe the behavior of real materials, introduced the assumption that the causes for brittle and ductile rupture occur independently. It is possible that deterioration and flow might change the values of n, C, and υ, however, this effect is not considered. Under this assumption the simplest method of combining the two theories would be to use Figure 1, and consider the smaller time to rupture at any stress level. A more rigorous approach would give the curve shown in Figure 1, (abe). Odqvist (1966, 1964) utilized a different expression for creep strain which resulted in lowering the stress-time to rupture curve to below the Kachanov-Hoff curve. This is also indicated schematically in Figure 1.

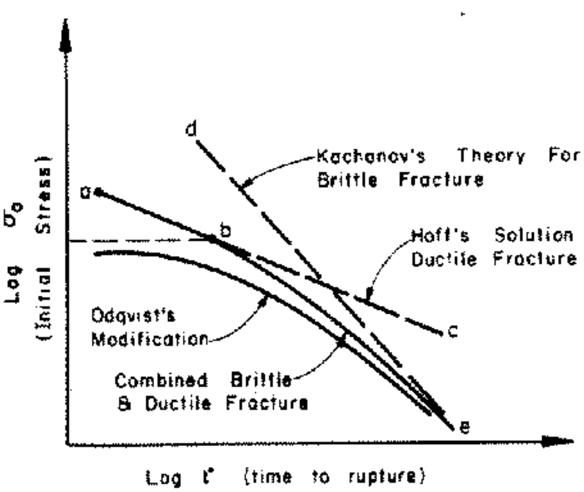

Figure 1. Failure theories on a logarithmic plot.

Creep rupture with varying stress

Theories of creep rupture under varying stress are based on the linear cumulative damage law. Such a law was first applied to creep rupture by Robinson (1951). It has been pointed out by Odqvist (1966) that Kachanov's theory of deterioration is also based on the linear accumulation of damage. Taira (1964) utilized similar concepts and considered the failure hypothesis of accumulated damage as a "life consumption hypothesis." Under such hypotheses, fracture is assumed to occur when the damage Φ_c accumulated in the material through creep reaches a critical value Φ_o.

Assume that a material subjected to a constant stress at a constant temperature ruptures at a time t_{rc}. Rupture occurs when the amount of accumulated damage Φ_c reaches the critical values of Φ_c. The amount of damage in a short interval of time Δt may be expressed in accordance with the following linear relation:

$$\Delta\phi_c = \phi_o \frac{\Delta t}{t_{rc}} \tag{14}$$

If it is assumed that load history has a negligible influence on creep rupture under varying stress, the amount of damage absorbed until time t under a varying stress can be written as:

$$\phi_c = \phi_o \int_0^t \frac{dt}{t_{rc}} \tag{15}$$

where t_{rc} is a function of the stress variation. Fracture (rupture) is assumed to occur when:

$$\phi_c/\phi_o \geq 1.0, \text{ i.e., } \int_0^t \frac{dt}{t_{rc}} \geq 1.0 \tag{16}$$

Suppose $\sigma = \sigma(t)$. If this variation is expressed as a stepwise function where stresses σ_1 to σ_n, which have times to rupture t_{rc1} to t_{rcn}, act for intervals Δt_1, to Δt_n. Then the damage that accumulates during this variation of stress can be written as:

$$\frac{\phi_c}{\phi_o} = \frac{\Delta t_1}{t_{rc1}} + \frac{\Delta t_2}{t_{rc2}} + \ldots \frac{\Delta t_n}{t_{rcn}} \tag{17}$$

For increased accuracy the number of steps in approximating the stress variation can be increased. Such a scheme of calculation is ideally suited for programming on a digital computer and is shown in Figure 5.

Creep rupture under multiaxial stress conditions

In almost all practical problems, multiaxial stress conditions exist. All the theories for creep rupture have been developed for uniaxial stress conditions. The generalization of these theories to multiaxial stress conditions is complicated and there is no general agreement on how this should be done. The essential point, in extending the uniaxial concepts discussed earlier to multiaxial conditions, is to establish a controlling stress. Once this is done, the selected controlling stress replaces the uniaxial stress and all the equations in the previous section remain valid.

The first and still most widely used theory was proposed by Kachanov where the uniaxial stress is replaced by the maximum principal tensile stress. For some materials, it has been proposed that the principal stress can be replaced by the effective stress. Odqvist (1966) points out that such a criteria should be considered in conjunction with some restrictions on the magnitude of the first stress invariant $1/3\ \sigma_{kk}$ i.e., that it be tensile. It has been hypothesized by Voorhees and Freeman (1959) that the tensile stress controls crack initiation and that the effective stress controls crack propagation.

It should be noted that in developing the theories of creep rupture, the essential condition was the stretching of the cylindrical specimen which caused a reduction in the area supporting the load, ultimately resulting in failure. It is possible under multiaxial stress conditions to cause tensile strains in one of the principal directions without any of the principal stresses being tensile. Therefore, it is suggested that requirements for multiaxial states of stress as suggested above may not be completely general. It is hypothesized that it is necessary to require that tensile strain occur in the material in one of the principal directions for creep rupture to occur.

Propagation of creep rupture under nonhomogeneous states of stress

An important consideration in the design of structures which are, in general, subject to nonhomogeneous states of stress is the propagation of the creep rupture surface in the material. A method of accounting for such phenomena has been proposed by Kachanov (see Odqvist (1964)). At some point in a material, the stress condition will be such that rupture can be assumed to have occurred at that point. Assume it occurs at t_{R1}. If the time to final rupture; i.e., when $\psi = 0$ for the entire structure, is t_{RZ}, the period $t_{R1} \leq t \leq t_{RZ}$ is called the state of the propagation of the failure. Kachanov has deduced an equation for the progress of the failure front. Some simple examples have been solved using this approach; however, further development will be required before it is applicable to problems in the analysis of structures in rock. At the present time, iterative techniques used in conjunction with numerical procedures for stress analysis and failure criteria are utilized to evaluate the possibility of failure propagation in a rock mass. These same techniques can be utilized when failure is assumed to occur due to creep rupture.

SUMMARY

Theories of creep rupture and their experimental verification have been conducted primarily for uniaxial stress states. These have been discussed above. The validity of creep rupture theories for multiaxial stress states has not been completely resolved at the present time. In addition, for complicated stress analysis problems, the difficulties in developing theories for varying and nonhomogeneous stress states have not been completely overcome. For practical problems in rock mechanics where the stress range is comparatively small and the experimental evidence is based on cored samples, a straight line relation between a single controlling stress*, and time to rupture is considered to be satisfactory approximation. At the present time, considering the stress conditions at the face of an excavation, the use of a single controlling stress in conjunction with a linear damage theory is one method that can be utilized for predicting failure in the practice of rock mechanics. There are no satisfactory theoretical methods to account for the propagation of creep rupture in case of gradual excavations in rock. Approximate techniques have to be utilized.

EXPERIMENTAL RESULTS

A few preliminary results from creep rupture tests under triaxial extension test conditions are presented in this paper. It is considered appropriate to discuss briefly the applicability of the triaxial extension test to stress conditions in the vicinity of an excavation. Figure 2 indicates schematically the stress conditions close to the face of an excavation in rock. One of the principal stresses is close to zero where as the other two principal stresses are nearly equal. Under these conditions, the rock face moves in towards the center of the opening. The rock close to the face is in extension in the radial direction. This extension increases with time and can cause spalling. It may be assumed that the stress difference $(\sigma_3 - \sigma_1)$ controls failure for spherically symmetric problems. For non-spherically symmetric problems, it is suggested that the maximum principal stress difference be utilized.

The stress conditions in a triaxial extension test are shown in Figure 3. It can be seen that the stress conditions are identical to the conditions in element A in Figure 2. For various combinations of σ_3/σ_1, the axial strain will be tensile. The deformation pattern is therefore similar to that which occurs in the rock near the face of the excavation. In the triaxial extension test, it is assumed that the stress difference $(\sigma_3 - \sigma_1)$ is the controlling stress and that the axial strain can be related to this stress.

Triaxial extension creep tests were conducted to determine the creep rupture characteristics of rock salt. Three important factors in the experimentation should be noted. Firstly, the tests were constant load tests. Consequently, the results are likely to be conservative. Secondly, in the loading history of the test, the specimen was first brought to equilibrium under an all round stress equal to the axial stress and then the radial stress was increased to obtain the desired value of the stress difference. In actual practice during the development of stress conditions in the rock

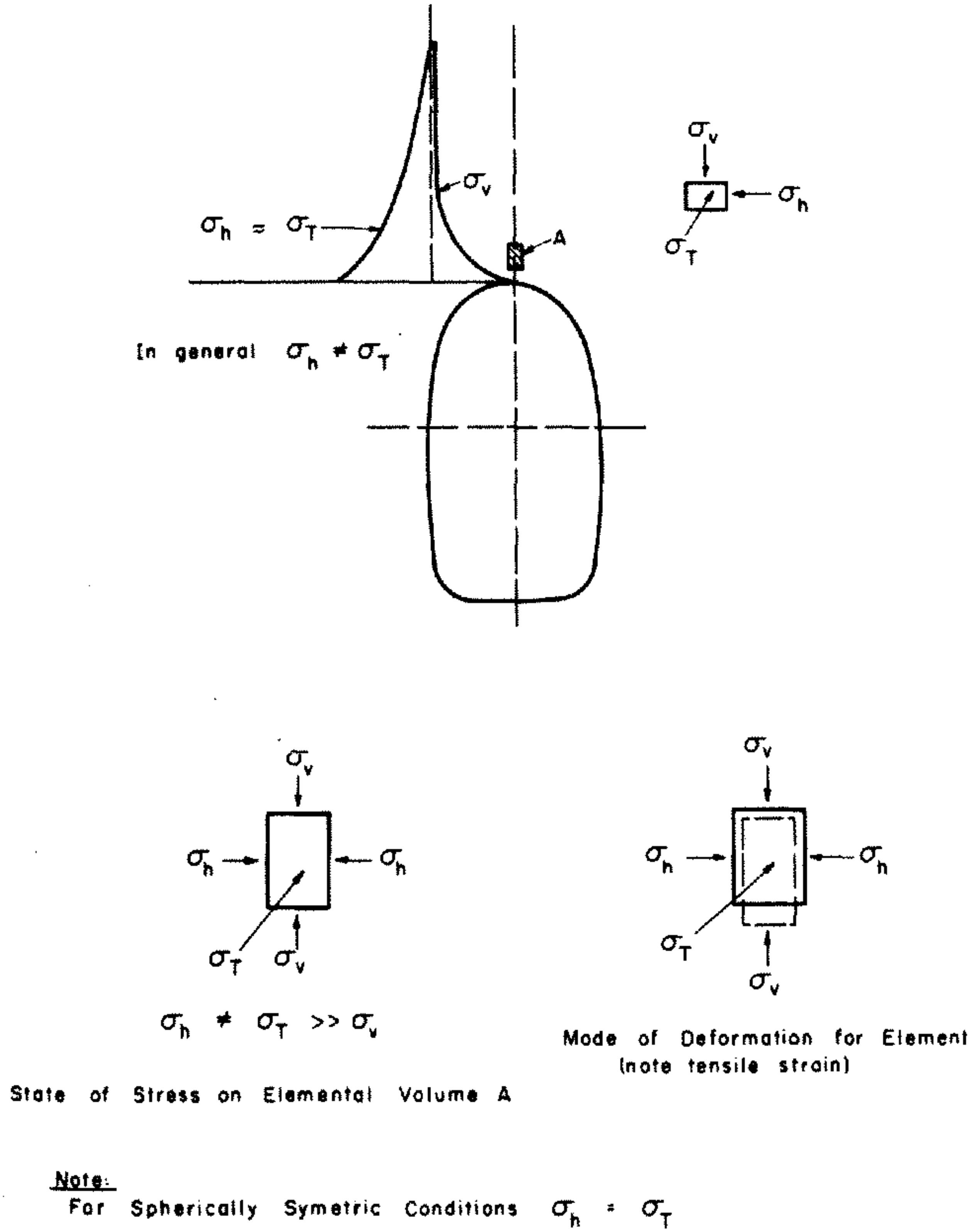

Figure 2. Stress and deformation conditions around an underground opening.

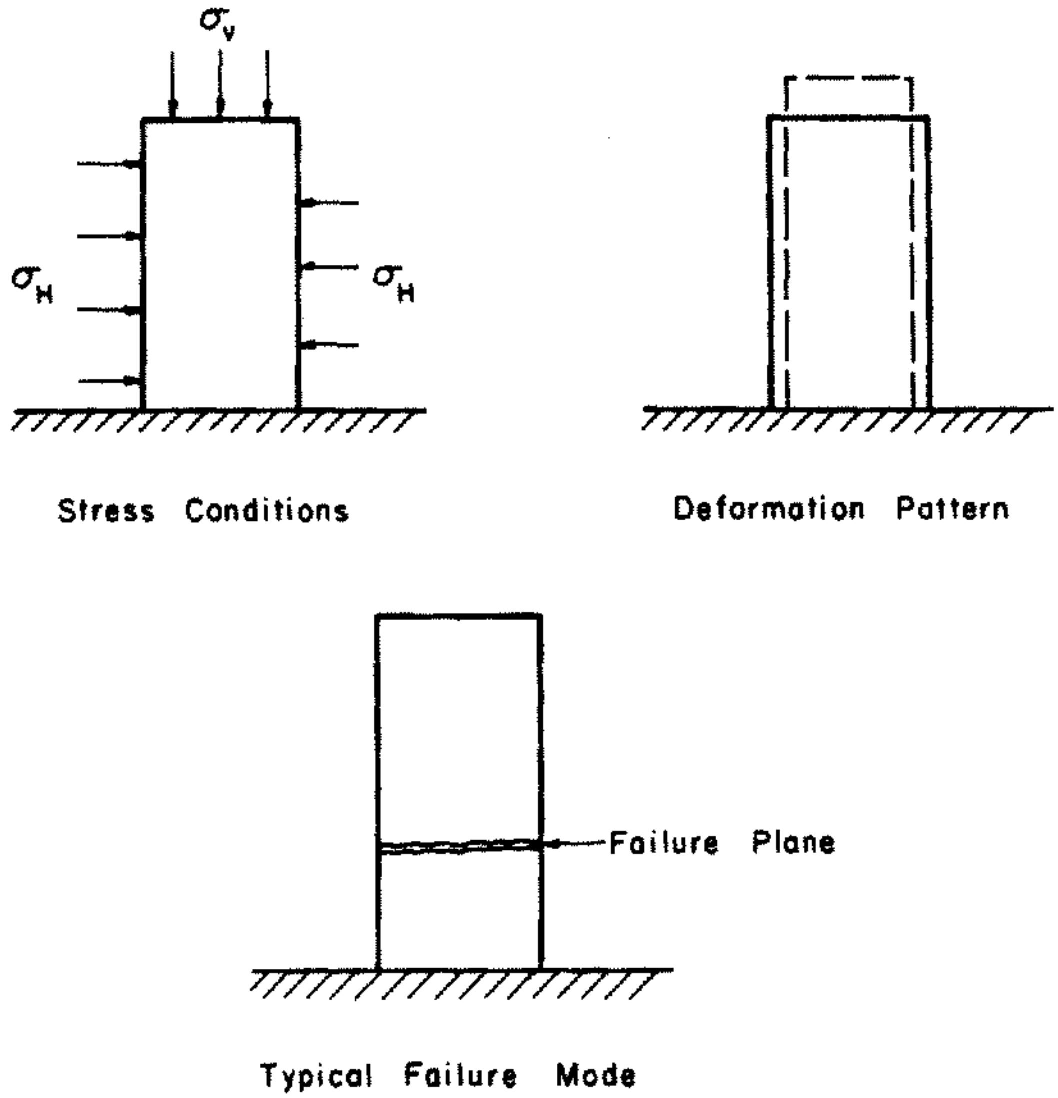

Figure 3. Stress, deformation and failure conditions in a triaxial extension test.

near an excavation face the existing equilibrium stress condition is disturbed by reducing one component of stress and increasing the others. The effect of load history has been investigated in a limited program of tests reported herein. Thirdly, all tests were conducted at a temperature of 68° F± 2°F. The effects of temperature on time to failure may be significant. Figure 3 indicates the mode of failure that occurred in the salt specimens. It can be seen that this mode of failure can be considered representative of the failure likely to cause spalling at the face of excavation in rock.

Creep rupture theories, discussed in the earlier part of this paper, suggest that stress-time rupture data can be plotted as a straight line on a logarithmic scale. The available data is plotted in Figure 4. The condition of zero axial stress is a special condition and is not considered in the plot data in Figure 4. The data obtained can be adequately represented by straight lines on a logarithmic plot shown in Figure 4. The values of first invariant of stress for the tests were from 5300 to 9700 psi. The data presented does not indicate any dependence of creep rupture time on the first invariant of stress. However, the tests represented here do not include any low stress tests. At low stresses, salt behaves as a Mohr Coulomb material and first invariant of stress is expected to have some effect. At higher pressures, applied in tests presented here, salt behaves as a Von Mises material and first invariant of stress will not affect the creep behavior. In boundary value problems, generally higher pressures are encountered and material behaves as a Von Mises material. For such problems a straight line as shown in Figure 4 is valid.

Experimentation

The test program consisted mainly of conducting creep tests under the following two conditions:

1. Triaxial extension creep tests where the stress condi-

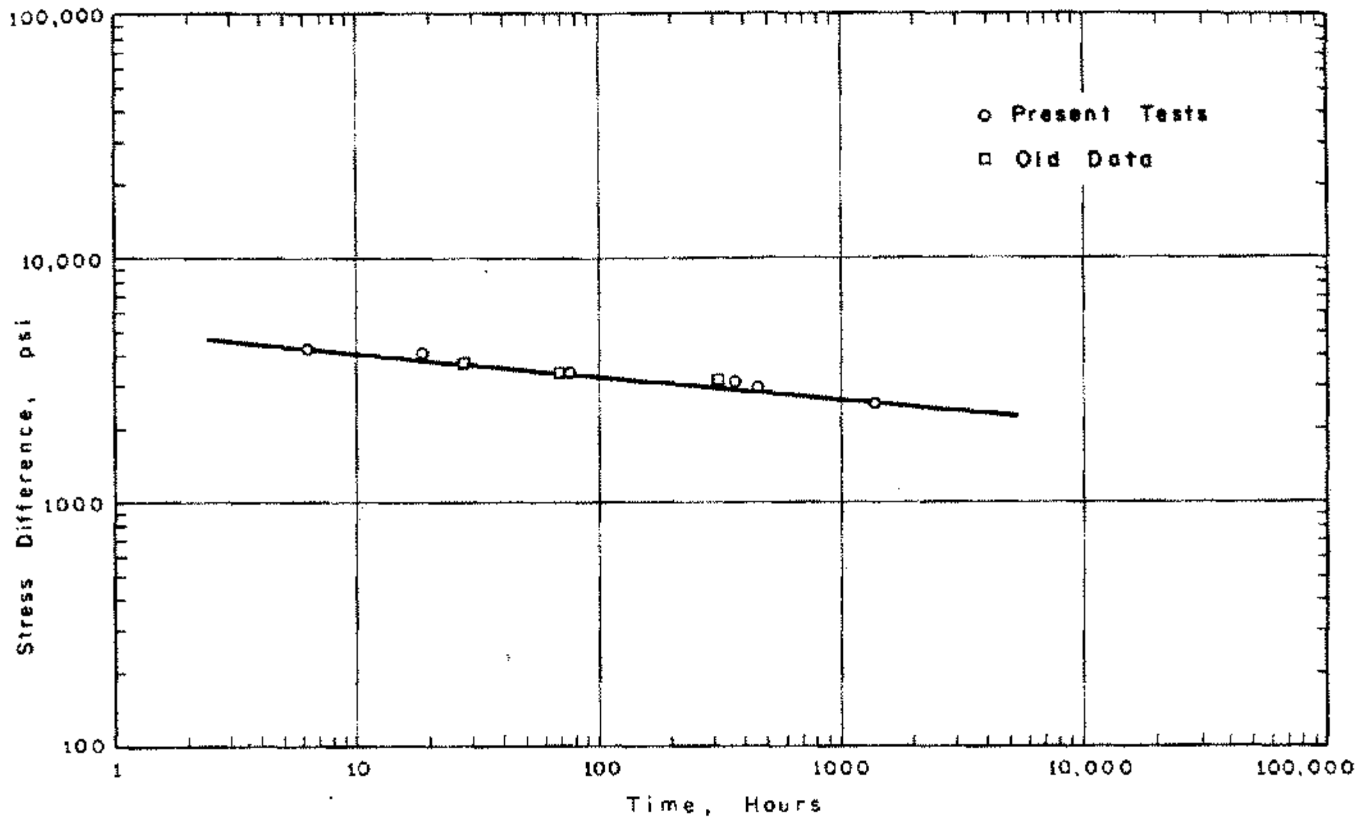

Figure 4. Stress difference—time to rupture.

tion at commencement was isotropic. The axial stress was then reduced. These tests attempted to relate the time for creep rupture to the deviatoric stress which was applied. The test results are shown in Figure (4). The time to rupture-deviatoric stress plot is a straight line on a log log plot. This is what is expected on the basis of Equation (11).

2. Triaxial extension creep tests with stepwise loading were conducted. In these tests the load was applied for some time and then increased again. The last increment was held till the specimen failed due to creep rupture. This was done to verify the applicability of Equations (13) and (17) to predict the time to rupture. An attempt was made to predict the time to rupture under the last load, given the loading history. It may be pointed out that at this stage only loading was considered. No consideration was given to unloading which gives the material time to recover.

Test no. 1. In this test, the sample was allowed approximately 24 hours to come to equilibrium under an all round pressure of 500 psi. No change in deformation was observed after this time. The sample was then subjected to the following stress history.

Stresses *Axial*	*Radial*	*Time (Minutes)*
500	2500	0–2958
500	3000	2958–5918
500	3500	5918–9880
500	4500	9880–10380

The total time to rupture was 10380 minutes and axial strain at failure 6.9%.

Test no. 2. In this test, the sample was allowed to come to equilibrium under an all round pressure of 2500 psi for a day. The sample was then subjected to the following stress history:

Stresses *Axial*	*Radial*	*Time (Minutes)*
0	2500	0–27000
0	3000	27000–52300

The total time to rupture was 52300 minutes and the axial strain at failure 3.1%.

Analysis of multiloading tests

The two tests were analyzed using Equations (13) and (17). An attempt was made to predict the time of rupture under the last increment of load, given the stress history of the sample. The following results were obtained:

Test No.	*Observed Time (Hours)*	*Equation (13)*	*Equation (17)*
1	8.33	10.6	9.8
2	420	222	180

These results show that observed and calculated times are of the same order. It may be pointed out that this predicts the time of rupture under the last increment where the error will accumulate due to experimental details. No attempt was made to take the stress concentrations into account. It is premature to conclude the validity or otherwise of Equations (13) and (17) or to prefer one over the other. But this shows that at least a beginning has been made, and it has been demonstrated that these equations can be used to predict creep of rupture for the two tests analyzed. Need for more tests on uniform material cannot be overemphasized.

INCORPORATION OF CREEP RUPTURE CRITERIA IN STABILITY ANALYSIS

A detailed discussion on the use of creep rupture criteria in investigating the stability of underground excavation is outside the scope of this paper. However, an outline of the methodology is presented herein as a series of steps.

1. Perform a stress analysis to determine the stress and displacement field in the rock mass surrounding the excavation. When creep effects are included the stress and displacement field will be obtained as functions of time.

2. Select a controlling stress and plot it as a function of time for certain selected locations in the vincinity of the cavity. This is shown schematically in Figure 5. The effective stress or the maximum principal stress difference might be used.

3. Approximate the continuous curve with a series of stress steps ($\sigma_1 \ldots \sigma_n$) as shown in Figure 5. For each of these steps stress acts for a certain time level, $\Delta t_{rc1} \ldots t_{rcn}$.

4. Based on laboratory test data, a stress-time rupture curve is established; e.g., Figure 4. From this, for each of stress steps ($\sigma_1 \ldots \sigma_n$) times to rupture $t_1 \ldots t_n$ are determined.

5. Knowing $\sigma_1 \ldots \sigma_n$, $t_1 \ldots t_n$ and Δt_{rcn}, it is possible to apply Equation 17 and determine if

$$\frac{\phi_c}{\phi_o}$$

is greater than 1.0, i.e., if failure is going to occur.

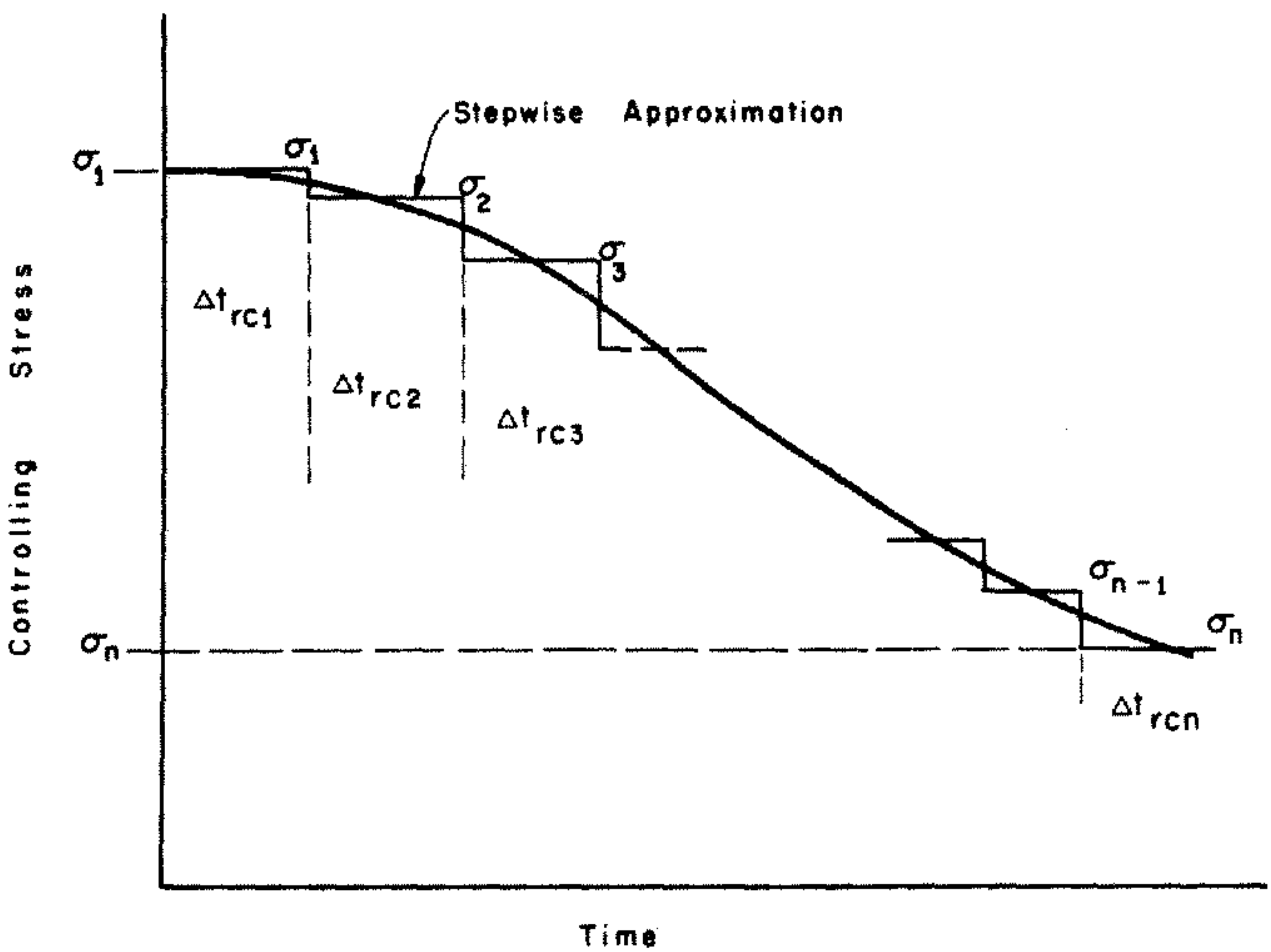

Figure 5. Stepwise approximation of continuous controlling stress–time curve.

Steps (1) to (5) are repeated for various locations. If failure is found to occur then the material at that location can be disregarded in future computations and the shape of the underground excavation will change. These repeated computations can be accomplished through the use of various iterative techniques.

FINAL REMARKS

It has been shown that creep rupture is a mode of failure that is likely to occur in rocks which undergo substantial time dependent deformation. On the basis of a theoretical review, which is supported by the limited experimental data available, it may be concluded that the controlling stress-time to rupture relationship can be represented as a straight line on a logarithmic plot. For multiaxial states of stress, there is some ambiguity about selecting the controlling stress. It has been suggested that for the conditions adjacent to the face of an excavation as represented by the triaxial extension test, the controlling stress is the maximum principal stress difference.

It has been the practice to look for tensile stresses in rock surrounding an underground cavity as a potential source of trouble. It appears that the occurrence of tensile strain is also a critical factor. In a triaxial extension test, the applied stresses are all comprehensive; however, the resulting axial strain which is tensile causes the specimen to fail. Similar conditions are likely to occur at the face of underground excavations and could cause surface spalling.

Failure in the laboratory has been observed for constant load tests. The linear cumulative damage theory for failure under varying stresses has been investigated in a limited program of testing. At the present time, a linear cumulative damage theory offers the only practical means of evaluating the potential of creep rupture under varying stress. The methodology for incorporating cumulative damage concepts into available methods of stress analysis to evaluate the potential failure in the rock mass has also been indicated.

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